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MICHIGAN TECHNOLOGICAL UNIVERSITY
HOUGHTON, MICHIGAN

THE STUDY OF VIBRATIONS GENERATED BY
THE TRACKS OF TRACKED VEHICLES

Delivery Order 0005
Contract No. DAAE07-75-A-0508

For

U.S. Army Tank-Automotive Research
and Development Command
Warren, Michigan 48090

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THE STUDY OF VIBRATIONS GENERATED BY THE TRACKS OF TRACKED VEHICLES

By

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Prepared For

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STUDY OF VIBRATIONS GENERATED BY THE TRACKS OF TRACKED VEHICLES

ABSTRACT

The crew members of tracked vehicles are affected adversely by low frequency vibrations transmitted to the vehicle compartment from the vibrations occurring in the track. The noise and vibration level in the crew compartment can cause hearing damage and serious discomfort to the crew members resulting in serious degradation of efficiency. These noise and vibration are caused by the transmission of the vibrations occurring in the track as it leaves the rear road wheel and goes over the idler and engages the sprocket. The chordal action in various parts of the track corresponding to the resonance-type vibrations also contribute to the noise and vibration. These factors, therefore, indicate loss of energy generated by the engine in addition to the discomfort to the crew.

The ~~research work described in this report~~ is an analytical study of the vibrations generated by the track of tracked vehicles. A method of analysis is derived from the technique of receptance calculation. By this means, the ratio of displacement at the idler wheel support to a periodic force applied at the rear road wheel, as the track pads strike the road, is calculated. This ratio can be obtained with due regards to the various physical parameters describing the characteristics of the track configuration and the boundary conditions at the idler wheel support. Analysis of forces acting on the idler wheel support also yields results

describing favorable idler wheel configuration, compliance of idler arm, and the size of the track shoe assembly. Combination of these results can be used to predict optimum conditions under which the vibration of a prescribed frequency can be minimized.

The present analytical study is a preliminary work that should lead into more extensive experimental and field work involving actual vehicles operating under various terrain conditions.

Letter on file

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STUDY OF VIBRATIONS GENERATED BY THE TRACKS OF TRACKED VEHICLES

I INTRODUCTION

The noise level in the crew compartment of tracked vehicles, such as tanks and personal carriers, can at some speeds exceed the pressure level that causes hearing damage. Low frequency vibration in the crew compartment is also known to cause serious discomfort to the crew members resulting in serious degradation of efficiency. The noise and vibrations are generated by several different sources. Engine noise, noise from the cooling fans, and the vibrations transmitted to the vehicle compartment as the track shoes strike the ground at periodic intervals, all contribute to the noise and vibration. All of these vibration sources are velocity dependent.

Another important source of the noise and vibration is the chordal action in the track of the vehicle. A previous study showed that the primary excitation of the low frequency noise and vibration in the vehicle compartment occurred when the track left the rear road wheel and went over the idler and engaged the sprocket. Transverse component of the vibration of the track between the rear road wheel and the idler has been shown to be the primary source of vibration transmitted to the vehicle compartment.

The research work described in this report is an analytical study of the vibration generated by the tracks of tracked vehicles. This analytical study will form a preliminary work that should lead into more extensive experimental and field work involving actual vehicles operating under various terrain conditions.

II PROBLEM STATEMENT

As the track leaves the rear road wheel and goes over the idler and engages the sprocket, vibrations will occur in the track, in the sprocket, and in the idler. A previous study* has shown that the transverse modes of the vibrations in the track are transmitted to the vehicle compartment thereby affecting the performance of both the vehicle and its crew. Severe coupling may also occur between the vehicle compartment and the standing-wave type transverse vibrations in the upper part of the track.

The chordal action in the track is assumed to depend on many factors, including track tension, vehicle speed, the ratio of the idler wheel diameter to the length of the track pitch, the interaction of the track with the terrain, etc.

The study of interaction between the track and the terrain is outside the intended scope of the present research. Although some work has been done to reduce the level of noise and vibration by means of different track configurations, such as the double-pin type or a re-design of idler arms, the problem involving the basic source of vibration, namely the track itself, remains to be investigated.

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*Report No. MPG-178, "Noise and Vibration Reduction Program on the M-109 Self-Propelled Howitzer", Allison Division Army Tank-Automotive Plant, 1966.

The present research is to involve the initiation of an analytical investigation of the vibrations generated in the track. The future application of the results of such a study to the reduction of noise and vibration in tracked vehicles without drastic re-design of the vehicle mechanisms is seen as the end objective.

III ANALYTICAL APPROACH TO THE PROBLEM

The basic aim of this study is to conduct an analytical investigation to predict conditions for reduction of noise and vibration generated in the track. Such a study will ordinarily require development of appropriate differential equations for track configuration as it traverses the path between the road wheel and the idler. These equations must be developed with due regard to the track tension and the length of the track pitch in relation to the idler wheel diameter. Solutions for these equations must be obtained for vibrations in the track under as many different boundary conditions as possible at the rear road wheel and the tension idler. The constraint imposed on this study is to analyze the problem within a limitation of no drastic re-design of vehicle system.

The usual approach employed in the vibrational analysis of periodic systems proves to have limited application in the study of track vibration for the purpose stated above. Physical parameters, such as the track pitch, mass of a shoe assembly, the moment of inertia of the shoe, etc., are all interdependent and it becomes intractable to predict what effect any one of these parameters has on vibration independent of the other parameters.

One analytical approach that reduces this complication is that based on the technique of receptance calculation. This method makes it possible to calculate the ratio of displacement at the idler to the periodic force input at the rear road wheel, the transfer receptance. The coefficient of transfer receptance will be expressed in terms of various physical parameters describing the track configuration and the boundary condition at the idler wheel.

The analysis of the receptance coefficient will then enable one to predict conditions under which the displacement at the idler will be a minimum for a given periodic force input at the rear road wheel.

IV ANALYSIS OF TRACK VIBRATION

The aim of this analysis is to obtain a solution that relates a periodic force input at the rear road wheel to the corresponding displacement or force at the idler wheel. Such an analysis will enable one to predict conditions that will minimize the displacement at the idler wheel for different values of input-force at the road wheel. The analysis will be developed from the calculation of receptance coefficients.

4.1 Definition and Calculation of Receptances

A track is represented as a series of identical masses coupled together at the joints*. At equilibrium, the system may either be in a straight-line configuration or at some angle between the adjacent elements (See Figure 1)

Each element of the system can be represented by the mass unit as shown in Figure 2. The pin at the left end of each element has a restoring torsion constant K (nt-m/rad) when a moment of force is applied to the element by application of forces at either end.

Consider one such element. The term F and u represent the force and the displacement, respectively (See Figure 3). The element has two "direct receptances" and one "transfer receptance" defined as follows:

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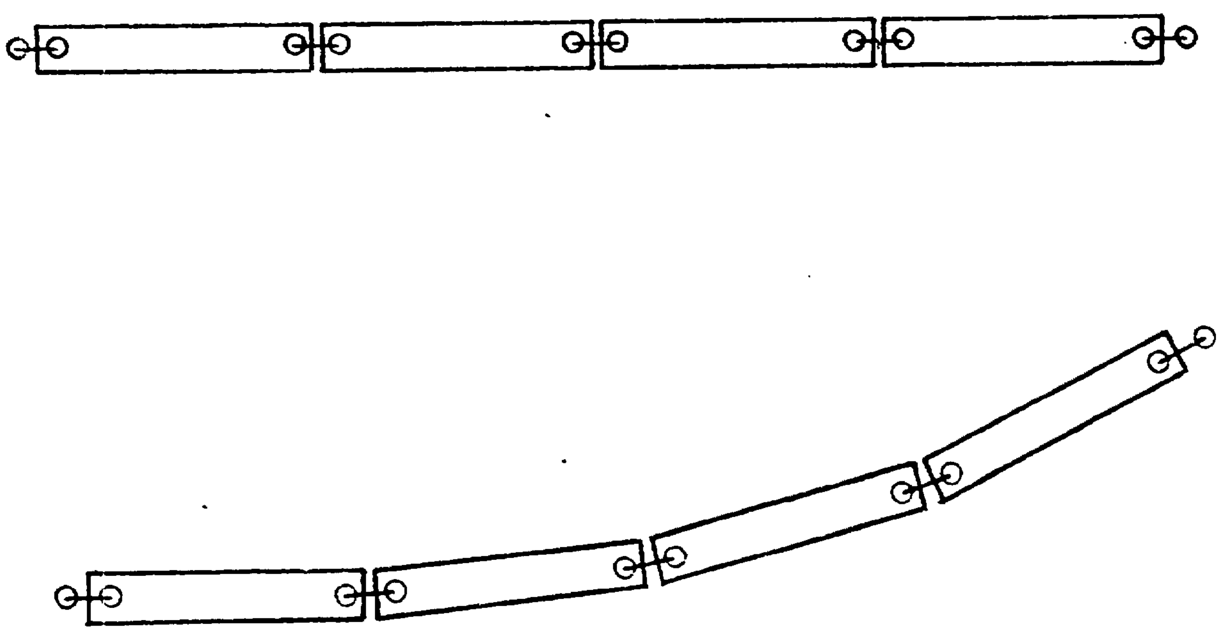
*The system treated here corresponds to a single-pin type track shoe assembly. For the double-pin configuration, the idealized model would be a series of alternating masses. The development of the basic theory in this case is essentially the same, but more complicated arithmetically.



(a)

(b)

Figure 1. Coupled periodic system representing a track. In equilibrium, the system may be in a straight-line configuration (a), or at some angle between adjacent elements (b).

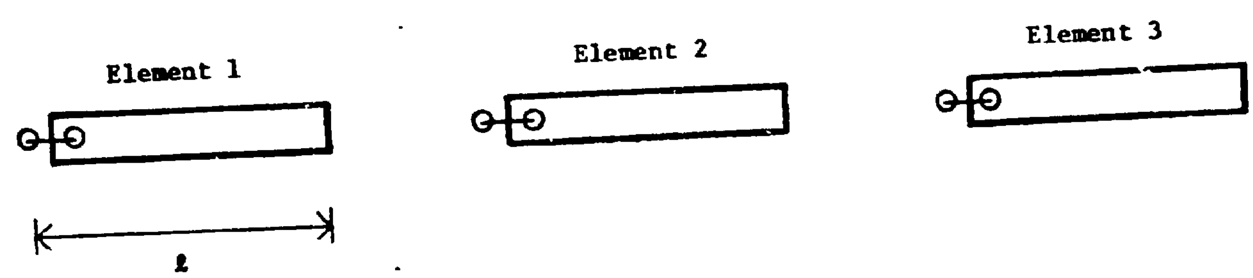


Figure 2. Elements of coupled masses.

Direct receptances:

$$\alpha_{11} = \frac{\text{Angular displacement corresponding to } u_1}{\text{Moment of force applied at 1 } (=F_1)}$$

$$= \frac{\text{Angular displacement about point 2}}{\text{Moment of force } F_1}$$

$$= \frac{u_1/\ell}{\ell F_1} = \frac{u_1}{\ell^2 F_1}, \qquad F_2 = 0 \tag{1}$$

$$\alpha_{22} = \frac{\text{Angular displacement corresponding to } u_2}{\text{Moment of force applied at 2 } (=F_2)}$$

$$= \frac{\text{Angular displacement about point 1}}{\text{Moment of force } F_2}$$

$$= \frac{u_2/\ell}{\ell F_2} = \frac{u_2}{\ell^2 F_2}, \qquad F_1 = 0. \tag{2}$$

Transfer receptance:

$$\alpha_{12} = \frac{\text{Angular displacement corresponding to } u_1}{\text{Moment of force applied at 2 } (=F_2)}$$

$$= \frac{\text{Angular displacement about point 2}}{\text{Moment of force } F_2}$$

$$= \frac{\text{Angular displacement corresponding to } u_2}{\text{Moment of force applied at 1 } (=F_1)}$$

$$= \frac{\text{Angular displacement about point 1}}{\text{Moment of force } F_1}$$

$$= \alpha_{21}. \tag{3}$$

Explicit expressions for these receptances can be calculated in the following manner. If a periodic force is applied at one end (F_1) and the system responds in a simple harmonic oscillation of frequency ω with no friction, then the equation of motion will yield

$$\ell F_1 = I\ddot{\theta} = -I\omega^2\theta = -I\omega^2 u_1/\ell \ , \tag{4}$$

where I is the moment of inertia of the element about 2. Therefore, from equations (1) and (4),

$$\alpha_{11} = u_1/\ell^2 F_1 = -1/I\omega^2 \tag{5}$$

for the direct receptance at 1.

If F_1 is applied and $F_2=0$, there is no angular distortion at the pin and $u_2=u_1$. Therefore,

$$u_2/\ell^2 F_1 = u_1/\ell^2 F_1$$

and

$$\alpha_{12} = u_2/\ell^2 F_1 = u_1/\ell^2 F_1 = -1/I\omega^2 = \alpha_{21} = \alpha_{11}. \tag{6}$$

For the calculation of α_{22}, consider the term u_2/ℓ.

$$\frac{u_2}{\ell} = \left(\begin{array}{c}\text{Angular displacement about 2} \\ \text{due to the moment applied by } F_2\end{array}\right)$$

$$+ \left(\begin{array}{c}\text{Angular displacement corresponding to the distortion of the pin} \\ \text{due to the moment applied by } F_2\end{array}\right)$$

$$= \alpha_{12}\ell F_2 + \ell F_2/K$$

$$= -\ell F_2/I\omega^2 + \ell F_2/K$$

$$= \ell F_2[(1/K) - (1/I\omega^2)] = \ell F_2[(I\omega^2 - K)/KI\omega^2]$$

and

$$\alpha_{22} = \frac{u_2/\ell}{\ell F_2} = \frac{I\omega^2 - K}{KI\omega^2} . \tag{7}$$

In the above definition and calculation of the receptances, it was implicitly assumed that there is no friction and the moment of inertia of each element is the same about either of its ends. In a real track assembly, the friction at the pin is likely to be much smaller in magnitude compared to the torsion constant and can be ignored in the first approximation. The moment of inertia of a shoe assembly will depend on a specific configuration of the shoe. Not much generality will be lost from the physics standpoint by assuming the same value of moment of inertia about either end of the element.

4.2 Wave Propagation in Periodic Elements

In order to investigate the propagation of disturbances in the track assembly as a periodic force is applied at one end, consider again the series of coupled elements, as shown in Figure 4. Each element is numbered by I, II, III, ..., and the ends of the element are identified by subscripts L (=left) and R (=right).

Since all the elements are identical, the phase relationship of the wave propagation between two different elements can be characterized by a propagation constant ε. Thus, for example, between two adjacent elements I and II,

$$F_{LII} = F_{LI}e^{-i\varepsilon}, \tag{8}$$

ε = propagation constant.

$$u_{LII} = u_{LI}e^{-i\varepsilon}, \tag{9}$$

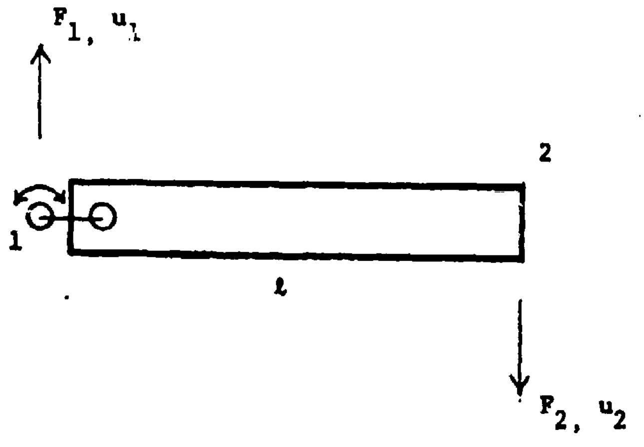


Figure 3. Forces and displacements from equilibrium at the ends of each element.

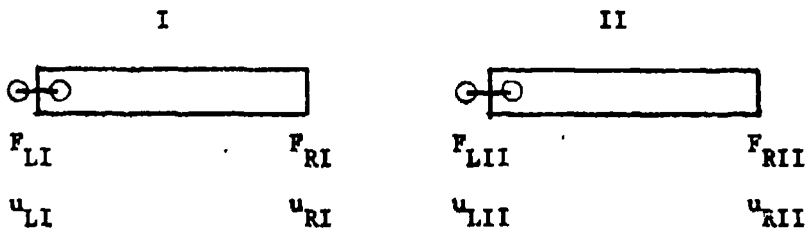


Figure 4. Forces and displacements at the ends of elements I, II, III, ... are identified by the subscripts L(=left) and R(=right).

Equilibrium condition requires that

$$F_{LII} = -F_{RI} \tag{10}$$

and the continuity condition requires that

$$u_{LII} = u_{RI}. \tag{11}$$

Therefore, combining these with equations (8) and (9),

$$F_{RI} = -e^{-i\epsilon}F_{LI}, \tag{12}$$

$$u_{RI} = e^{-i\epsilon}u_{LI}. \tag{13}$$

Now, from the definitions of the receptances

$$\begin{pmatrix} u_{LI} \\ u_{RI} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \ell^2 F_{LI} \\ \ell^2 F_{RI} \end{pmatrix} \qquad \alpha_{12} = \alpha_{21}$$

$$= \begin{pmatrix} \alpha_{11}\ell^2 F_{LI} + \alpha_{12}\ell^2 F_{RI} \\ \alpha_{21}\ell^2 F_{LI} + \alpha_{22}\ell^2 F_{RI} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{11}\ell^2 F_{LI} + \alpha_{12}\ell^2 F_{RI} \\ e^{-i\epsilon}\alpha_{11}\ell^2 F_{LI} + e^{-i\epsilon}\alpha_{12}\ell^2 F_{RI} \end{pmatrix}, \tag{14}$$

where, in the last step, a relationship (13) was used to write

$$F_{LI}\alpha_{21} + F_{RI}\alpha_{22} = u_{RI}/\ell^2 = e^{-i\varepsilon}u_{LI}/\ell^2$$

$$= e^{-i\varepsilon}(F_{LI}\alpha_{11} + F_{RI}\alpha_{12}).$$

Rewriting the above result by transferring the terms on the righthand side to the left and combining with equation (12), one obtains a two simultaneous equations for F_{RI} and F_{LI}:

$$\begin{cases} F_{RI}(\alpha_{22} - e^{-i\varepsilon}\alpha_{12}) + F_{LI}(\alpha_{12} - e^{-i\varepsilon}\alpha_{11}) = 0 \\ F_{RI} + e^{-i\varepsilon}F_{LI} = 0 \end{cases}$$

For non-trivial solution of the above equations, it is required that

$$\begin{vmatrix} \alpha_{22} - e^{-i\varepsilon}\alpha_{12} & \alpha_{12} - e^{-i\varepsilon}\alpha_{11} \\ 1 & e^{-i\varepsilon} \end{vmatrix} = 0,$$

or

$$(\alpha_{22} - e^{-i\varepsilon}\alpha_{12})e^{-i\varepsilon} - (\alpha_{12} - e^{-i\varepsilon}\alpha_{11}) = 0$$

from which it results that

$$\cos \varepsilon = \frac{\alpha_{22} + \alpha_{11}}{2\alpha_{12}} . \tag{15}$$

This expression gives the propagation constant ε in terms of the receptance coefficients.

4.2.1 Characteristic Receptances

In a periodic system, a wave disturbance can propagate in either direction. Let a single wave propagate through the periodic system. Then, from equations (12) and (14),

$$F_{RI} = -e^{-i\varepsilon} F_{LI},$$

$$u_{LI} = \omega^2 F_{LI}(\alpha_{11} - e^{-i\varepsilon}\alpha_{12}),$$

a receptance can be defined by

$$\alpha_{w+} = \frac{u_{LI}}{\omega^2 F_{LI}} = \alpha_{11} - e^{-i\varepsilon}\alpha_{12}. \qquad (16)$$

This expression defines the "characteristic wave receptance" of a positive-going wave in an infinite periodic system.

For the wave traveling in the opposite direction, the replacement of $e^{-i\varepsilon}$ by $e^{+i\varepsilon}$ will result in the characteristic wave receptance for the negative-going wave:

$$\alpha_{w-} = \alpha_{11} - e^{+i\varepsilon}\alpha_{12}. \qquad (17)$$

4.3 Receptances of a Finite Periodic System

The periodic system corresponding to a track configuration between the rear road wheel and the idler must be represented by a finite system with a suitable boundary. Such a system is shown in Figure 5.

Harmonic force F_0 is applied at the lefthand end A. The term u_0 is the displacement at A. The boundary B at the righthand end corresponds to the idler position, where the receptance is assumed to be α_B. There are N elements in the periodic system.

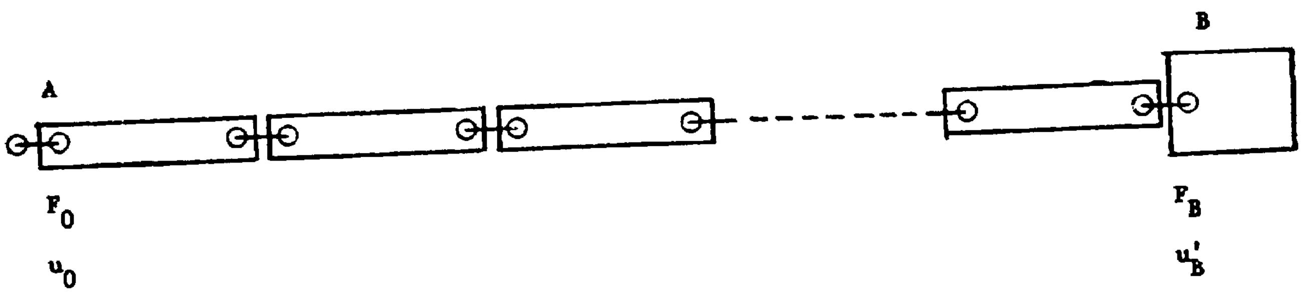


Figure 5. Coupled periodic system with a boundary at one end. The boundary B corresponds to the idler wheel and its support system in a track.

When a periodic force F_0 is applied at the road wheel, it generates a positive-going wave in the system whose displacement at A is u_+. The wave travels to the boundary B, is reflected and returns as a negative-going wave to A, where its contribution to the displacement is u_-. Therefore, the total displacement at A is

$$u_0 = u_+ + u_- . \tag{18}$$

Associated with these displacements are the forces

$$F_0 = F_+ + F_- , \tag{19}$$

where, with the help of characteristic receptances defined in equations (16) and (17), it can be written that

$$\begin{aligned} u_+ &= \alpha_{w+} F_+ \ell^2 , \\ u_- &= \alpha_{w-} F_- \ell^2 . \end{aligned} \tag{20}$$

Hence,

$$F_0 = \frac{u_+}{\ell^2 \alpha_{w+}} + \frac{u_-}{\ell^2 \alpha_{w-}} . \tag{21}$$

It is necessary to know F_- in terms of F_+. This is determined by the boundary conditions. At B, the total force due to the two wave components is

$$F_B = F_+ e^{-iN\varepsilon} + F_- e^{iN\varepsilon} , \tag{22}$$

that is, the force in the positive wave changes by ε per periodic element and, over the N elements to the boundary B, it changes phase by $N\varepsilon$. Similarly, the total displacement at B due to the two wave components is

$$u_B = u_+ e^{-iN\epsilon} + u_- e^{iN\epsilon} . \tag{23}$$

Now, the force F_B acts on the boundary B and causes the boundary to have a displacement u_B' given by

$$u_B' = \ell^2 F_B \alpha_B . \tag{24}$$

The displacement must be identical to the total displacement in the periodic system at B, that is, to u_B. Therefore,

$$\ell^2 F_B \alpha_B = u_+ e^{-iN\epsilon} + u_- e^{iN\epsilon}$$

or, using equations (20) and (22),

$$(\alpha_{w+} \ell^2 F_+) e^{-iN\epsilon} + (\alpha_{w-} \ell^2 F_-) e^{iN\epsilon}$$

$$= \alpha_B (\ell^2 F_+ e^{-iN\epsilon} + \ell^2 F_- e^{iN\epsilon}).$$

Hence,

$$F_+(\alpha_{w+} - \alpha_B) e^{-iN\epsilon} + F_-(\alpha_{w-} - \alpha_B) e^{iN\epsilon} = 0$$

or

$$F_- = -F_+ \left(\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B} \right) e^{-2iN\epsilon} . \tag{25}$$

From equations (19) and (25), the force at A is obtained as

$$F_0 = F_+ + F_-$$

$$= F_+ \left[1 - \left(\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B} \right) e^{-2iN\epsilon} \right] . \tag{26}$$

From equations (18) and (25), the displacement at A is obtained as

$$u_0 = u_+ + u_-$$

$$= \ell^2 F_+ \alpha_{w+} + \ell^2 F_- \alpha_{w-}$$

$$= \ell^2 F_+ [\alpha_{w+} - \alpha_{w-} (\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B}) e^{-2iN\epsilon}]. \tag{27}$$

4.3.1 Direct receptance and complex force reflection ratio

The direct receptance at A is defined as the ratio of the angular displacement corresponding to u_0 to the moment of force applied by F_0:

$$\alpha_{AA} = u_0 / \ell^2 F_0$$

$$= \frac{\alpha_{w+} - \alpha_{w-} (\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B}) e^{-2iN\epsilon}}{1 - (\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B}) e^{-2iN\epsilon}}$$

$$= \frac{\alpha_{w+} + \alpha_{w-} r_{fB} e^{-2iN\epsilon}}{1 + r_{fB} e^{-2iN\epsilon}} , \tag{28}$$

where

$$r_{fB} = - \frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B} = \frac{\ell^2 F_{B-}}{\ell^2 F_{B+}} = \frac{F_{B-}}{F_{B+}}$$

$$= \frac{\text{Force in the negative-going reflected wave at B}}{\text{Force in the incident wave at B}}$$

$$= \text{Complex force reflection ratio},$$

or, with the help of equations (16) and (17),

$$r_{fB} = -\frac{\alpha_{11} - e^{-i\varepsilon}\alpha_{12} - \alpha_B}{\alpha_{11} - e^{i\varepsilon}\alpha_{12} - \alpha_B} \quad . \tag{29}$$

Explicit expression for the direct receptance at A may be obtained by rewriting equation (28):

$$\alpha_{AA} = \frac{\alpha_{w+}(\alpha_{w-} - \alpha_B)e^{iN\varepsilon} - \alpha_{w-}(\alpha_{w+} - \alpha_B)e^{-iN\varepsilon}}{(\alpha_{w-} - \alpha_B)e^{iN\varepsilon} - (\alpha_{w+} - \alpha_B)e^{-iN\varepsilon}}$$

$$= \frac{\alpha_{w+}\alpha_{w-}(e^{iN\varepsilon} - e^{-iN\varepsilon}) - \alpha_B(\alpha_{w+}e^{iN\varepsilon} - \alpha_{w-}e^{-iN\varepsilon})}{(\alpha_{w-}e^{iN\varepsilon} - \alpha_{w+}e^{-iN\varepsilon}) - \alpha_B(e^{iN\varepsilon} - e^{-iN\varepsilon})} \; .$$

But,

$$e^{iN\varepsilon} - e^{-iN\varepsilon} = 2i \sin N\varepsilon \tag{30}$$

and, using equations (16) and (17),

$$\alpha_{w+}\alpha_{w-} = \alpha_{11}^2 + \alpha_{12}^2 - \alpha_{11}\alpha_{12}(e^{i\varepsilon} + e^{-i\varepsilon})$$

$$= \alpha_{11}^2 + \alpha_{12}^2 - 2\alpha_{11}\alpha_{12} \cos \varepsilon , \tag{31}$$

$$\alpha_{w+}e^{iN\varepsilon} - \alpha_{w-}e^{-iN\varepsilon} = \alpha_{11}e^{iN\varepsilon} - \alpha_{12}e^{i(N-1)\varepsilon} - \alpha_{11}e^{-iN\varepsilon} + \alpha_{12}e^{-i(N-1)\varepsilon}$$

$$= 2i[\alpha_{11} \sin N\varepsilon - \alpha_{12} \sin(N-1)\varepsilon] , \tag{32}$$

$$\alpha_{w-}e^{iN\varepsilon} - \alpha_{w+}e^{-iN\varepsilon} = \alpha_{11}e^{iN\varepsilon} - \alpha_{12}e^{i(N+1)\varepsilon} - \alpha_{11}e^{-iN\varepsilon} + \alpha_{12}e^{-i(N+1)\varepsilon}$$

$$= 2i[\alpha_{11} \sin N\varepsilon - \alpha_{12} \sin(N+1)\varepsilon] . \tag{33}$$

Therefore,

$$\alpha_{AA} = \frac{(\alpha_{11}^2+\alpha_{12}^2-2\alpha_{11}\alpha_{12}\cos\varepsilon)2i\sin N\varepsilon - 2i\alpha_B[\alpha_{11}\sin N\varepsilon - \alpha_{12}\sin(N-1)\varepsilon]}{2i[\alpha_{11}\sin N\varepsilon - \alpha_{12}\sin(N+1)\varepsilon] - 2i\alpha_B\sin N\varepsilon}$$

$$= \frac{(\alpha_{11}^2+\alpha_{12}^2-2\alpha_{11}\alpha_{12}\cos\varepsilon - \alpha_{11}\alpha_B)\sin N\varepsilon + \alpha_{12}\alpha_B\sin(N-1)\varepsilon}{(\alpha_{11}-\alpha_B)\sin N\varepsilon - \alpha_{12}\sin(N+1)\varepsilon}\,. \tag{34}$$

4.3.2 Transfer receptance

The term which is directly related to the present study is the ratio of the absolute displacement at B to the periodic force applied at A. This can be obtained from the calculation of the "transfer receptance" at B. The transfer receptance is defined as the ratio of the angular displacement corresponding to u_B to the moment of force applied by F_0.

From equations (20), (23) and (25),

$$u_B = u_+e^{-iN\varepsilon} + u_-e^{iN\varepsilon}$$

$$= \ell^2F_+\alpha_{w+}e^{-iN\varepsilon} + \ell^2F_-\alpha_{w-}e^{iN\varepsilon}$$

$$= \ell^2F_+[\alpha_{w+}e^{-iN\varepsilon} - \alpha_{w-}e^{iN\varepsilon}(\frac{\alpha_{w+}-\alpha_B}{\alpha_{w-}-\alpha_B})e^{-2iN\varepsilon}].$$

Combining this result with equation (27), one obtains

$$\alpha_{BA} = \alpha_{AB} = \frac{u_B}{\ell^2 F_0}$$

$$= \frac{\alpha_{w+}e^{-iN\varepsilon} - \alpha_{w-}e^{iN\varepsilon}\left(\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B}\right)e^{-2iN\varepsilon}}{1 - \left(\frac{\alpha_{w+} - \alpha_B}{\alpha_{w-} - \alpha_B}\right)e^{-2iN\varepsilon}}$$

$$= \frac{\alpha_{w+}(\alpha_{w-} - \alpha_B)e^{-iN\varepsilon} - \alpha_{w-}(\alpha_{w+} - \alpha_B)e^{iN\varepsilon}e^{-2iN\varepsilon}}{(\alpha_{w-} - \alpha_B) - (\alpha_{w+} - \alpha_B)e^{-2iN\varepsilon}}$$

$$= \frac{\alpha_{w+}\alpha_{w-} - \alpha_{w+}\alpha_B - \alpha_{w-}\alpha_{w+} + \alpha_{w-}\alpha_B}{(\alpha_{w-}e^{iN\varepsilon} - \alpha_{w+}e^{-iN\varepsilon}) - \alpha_B(e^{iN\varepsilon} - e^{-iN\varepsilon})}$$

$$= -\frac{(\alpha_{w+} - \alpha_{w-})\alpha_B}{2i[\alpha_{11}\sin N\varepsilon - \alpha_{12}\sin(N+1)\varepsilon] - 2i\alpha_B \sin N\varepsilon} .$$

But

$$-(\alpha_{w+} - \alpha_{w-})\alpha_B = \alpha_{12}(e^{-i\varepsilon} - e^{i\varepsilon})\alpha_B$$

$$= -2i\alpha_{12}\alpha_B \sin \varepsilon .$$

Therefore, the transfer receptance at B takes the expression

$$\alpha_{BA} = -\frac{\alpha_{12}\alpha_B \sin \varepsilon}{(\alpha_{11} - \alpha_B)\sin N\varepsilon - \alpha_{12}\sin(N+1)\varepsilon} . \tag{35}$$

4.4 Analysis of Transfer Receptance

The ratio of the displacement at B to the force applied at A is obtained from the transfer receptance, equation (35), as

$$\frac{u_B}{F_0} = \ell^2 \alpha_{BA}$$

$$= -\frac{\ell^2 \alpha_{12} \alpha_B \sin \varepsilon}{(\alpha_{11} - \alpha_B)\sin N\varepsilon - \alpha_{12}\sin(N+1)\varepsilon} , \qquad (36)$$

where

$$\alpha_{11} = -\frac{1}{I\omega^2} ,$$

$$\alpha_{12} = \alpha_{21} = -\frac{1}{I\omega^2} = \alpha_{11} ,$$

$$\alpha_{22} = \frac{I\omega^2 - K}{KI\omega^2} = \frac{1}{K} - \frac{1}{I\omega^2} = \frac{1}{K} + \alpha_{11} = \frac{1}{K} + \alpha_{12} = \frac{1}{K} + \alpha_{21}, \qquad (37)$$

$$\cos \varepsilon = \frac{\alpha_{22} + \alpha_{11}}{2\alpha_{12}} = \frac{(1/K)+2\alpha_{11}}{2\alpha_{11}} = 1 + \frac{1}{2K\alpha_{11}} = 1 - \frac{I\omega^2}{2K} .$$

The expression (36) shows the ratio of the displacement at the idler wheel position due to the force applied at the rear road wheel as the wheel strikes the road surface. At constant speeds F_0 will be a periodically applied force whose frequency is proportional to the speed. The purpose of this calculation is to find the condition in which the ratio u_B/F_0 becomes a minimum for a given frequency ω. The ratio u_B/F_0 depends on several physical parameters, I, K, N, and ℓ, which can be varied one at a time while the others are held fixed, or more than one at a time. A few preliminary results are immediately obtained as shown below.

4.4.1 Receptance of B

The receptance α_B at B corresponds to the boundary condition at the idler wheel suspension. Two extreme boundary conditions are those for a free end and for a fixed end.

If free-ended, $\alpha_B \to \infty$ and

$$\frac{u_B}{F_0} \to \frac{\ell^2 \alpha_{12} \sin \varepsilon}{\sin N\varepsilon} . \tag{38}$$

If fixed-end, $\alpha_B \to \infty$ and

$$\frac{u_B}{F_0} \to 0. \tag{39}$$

These two extreme boundary conditions show that the more rigid the boundary B is, the smaller the value of u_B for a given input force F_0 at A. A completely rigid end corresponds to an idler suspension with zero compliance; in this case, the vehicle body and the idler wheel would move as a single mass unit, describing a physically impractical situation of no idler wheel at all. Nevertheless, this result is in agreement with the result described in the GM report, referred to earlier, that showed the running of a tank with no idler wheel actually reduced the internal vibration in the crew compartment.

The complex force reflection ratio r_{fB}, equation (29), is

$$r_{fB} = - \frac{\alpha_{11} - e^{-i\varepsilon}\alpha_{12} - \alpha_B}{\alpha_{11} - e^{i\varepsilon}\alpha_{12} - \alpha_B} = - \frac{1 \quad e^{-i\varepsilon} - \alpha'}{1 - e^{i\varepsilon} - \alpha'} , \tag{38}$$

where

$$\alpha' = \alpha_B/\alpha_{11} = \alpha_B/\alpha_{12} = \text{relative compliance.} \tag{39}$$

The relative compliance defined above can take values between 0 and ∞, corresponding to the two extreme boundary conditions of fixed-end and free-end, respectively. The reflection ratio r_{fB}, which must be less than one, increases with decreasing α_B. Thus, as $\alpha_B \to 0$, the force in the negative-going reflected wave at B tends to increase. Therefore, while the value of u_B decreases as $\alpha_B \to 0$, the chordal action in the track between the rear road wheel and the idler wheel will have a greater amplitude, thereby causing a severe vibration in the track itself. This will cause a greater energy loss.

In practice, a desirable balance may be achieved by making an idler arm compliance relatively stiff (α_B must be small but not zero) at the same time increasing the mass of the idler wheel. In short, the bigger the idler wheel, the less vibration is transmitted to the vehicle compartment through the idler.

4.4.2 Size of Track Shoe Assembly

The size of the track shoe, ℓ, and the number of the shoe assembly, N, between the road wheel and the idler wheel are dependent on each other. If this dependence is ignored for the moment, the smaller value of ℓ corresponds to the smaller value for u_B/F_0 since $u_B/F_0 \propto \ell^2$ from equation (36).

Thus, less vibration will be transmitted to the vehicle if the shoe assembly is made small. When this result is combined with that in 4.4.1, namely that for a larger idler wheel, the optimum condition will be reached at the "continuous-belt" limit of the track.

4.4.3 Numerical analysis of transfer receptance

Since $\alpha_{11} = \alpha_{12}$, the ratio u_B/F_0 in equation (36) may be rewritten as

$$\frac{u_B}{F_0} = -\frac{\ell^2\alpha_{11}\alpha_B \sin\varepsilon}{\alpha_{11}[\sin N\varepsilon - \sin(N+1)\varepsilon] - \alpha_B \sin N\varepsilon}$$

$$= -\frac{\ell^2\alpha_B \sin\varepsilon}{\sin N\varepsilon - \sin(N+1)\varepsilon - \alpha' \sin N\varepsilon}, \qquad (40)$$

where α' is defined in equation (39). Even though α' may take any value between zero (fixed-end) and infinity (free-end), the reasonable practical range of values may be assumed to be 0.01 ~ 0.25. For real vehicles, the value of α' must be determined from the measurement of α_B at the idler wheel. This measurement may present a difficult experimental problem.

From equation (37),

$$\cos\varepsilon = 1 - \frac{I\omega^2}{2K} = 1 + \frac{1}{2K\alpha_{11}} .$$

Since $-1 \leq \cos\varepsilon \leq 1$, the above expression yields

$$0 \leq \frac{I\omega^2}{K} \leq 4, \qquad (41)$$

and

$$\sin\varepsilon = (1 - \cos^2\varepsilon)^{1/2} = [(I\omega^2/K) - (I\omega^2/2K)^2]^{1/2}. \qquad (42)$$

Inspection of equation (40) shows that the ratio u_B/F_0 is not a smooth function. This is evident from the fact that the three terms occurring in that equation, $\sin\varepsilon$, $\sin N\varepsilon$, and $\sin(N+1)\varepsilon$, are all smooth sinusoidal functions of different frequencies and that a ratio of combination of these terms will not generally yield a smooth function. In order to find conditions in which u_B/F_0 becomes a minimum it is most convenient to use a numerical analysis of equation (40).

The procedure to follow in the numerical analysis is as follows: For a given frequency ω (that is, for a given vehicle speed), vary the value of $I\omega^2/K$ between zero and four. The term $I\omega^2/K$ for a given value of ω indicates the ratio of the moment of inertia of a shoe assembly to the torsion constant of the pin. Corresponding to each value of $I\omega^2/K$ is a value for sin ϵ, or equivalently, for ϵ.

Next, choose a value for α' and a value for N, the number of shoe assembly between the rear road wheel and the idler wheel. Using these values in equation (40), the ratio u_B/F_0 may be calculated and tabulated for evaluation.

Appendix A shows a few sample calculations for several typical values of α' and for $N = 5\text{~}9$.

The result confirms that the ratio u_B/F_0 is not a smooth function, in general. The tables also show some drastic conditions that should be avoided if one wants to minimize vibrations transmitted through idler. For example, $N=5$, $\alpha'=0.15$ and $I\omega^2/K=1.75$ are a bad combination of parameters, while a slight alteration of the condition by increasing N by one drastically reduces the ratio $(1/\ell^2\alpha_B)u_B/F_0$.

The sample calculations in Appendix A contain two different cases.

(i) The case in which ℓ is independent of N. This would correspond to the case in which the distance between the rear road wheel and the idler wheel is not fixed.

The dependence of u_B/F_0 on N for a fixed value of ℓ may be computed by non-dimensionalizing the equation (40), thus:

$$\left(\frac{1}{\ell^2 \alpha_B}\right) \frac{u_B}{F_0} = - \frac{\sin \epsilon}{(1 - \alpha')\sin N\epsilon - \sin(N+1)\epsilon} = -B. \tag{43}$$

(ii) The case in which $N\ell = L$ = constant. This would correspond to the case in which the distance between the rear road wheel and the idler wheel is fixed. Therefore, ℓ can be made smaller only by increasing N, and vice versa. In this case

$$\left(\frac{1}{L^2 \alpha_B}\right) \frac{u_B}{F_0} = - \frac{(\sin\epsilon)/N^2}{(1 - \alpha')\sin N\epsilon - \sin(N+1)\epsilon} = \frac{-B}{N^2} . \tag{44}$$

These two cases are tabulated in separate tables in Appendix A. The calculations shown are only for the purpose of illustration. Similar tables may be constructed for different cases. The calculations are simple enough to be programmed in a hand-held calculator.

V ANALYSIS OF FORCES ACTING ON THE IDLER WHEEL SUPPORT

The forces acting on the idler wheel support consist of three types, two of which are static and the other dynamic. They are

(i) Static tension of the track,

(ii) Gravitational force of

a. idler wheel mass M_I,

b. part of the track contributing to the weight on the wheel support bearing,

(iii) Dynamic effect of centripetal force on the track.

The static forces (track tension and gravitational forces) will not contribute directly to the vibration coupling between the track and the vehicle compartment. The dynamic effect of the centripetal force on the track as it goes around the idler will be a contributing factor to the vibration coupling.

5.1 Static Tension of the Track

The figure shown, Figure 6, illustrates the tension force applied on the idler wheel support. At constant speeds, $T_1 = T_2$ and it will not contribute to the vibration coupling.

5.2 Gravitational Force

The contribution to the forces acting on the idler wheel support due to gravitation comes from

a. the mass of the idler wheel M_I,

b. the mass of the part of track contributing to the weight at C.

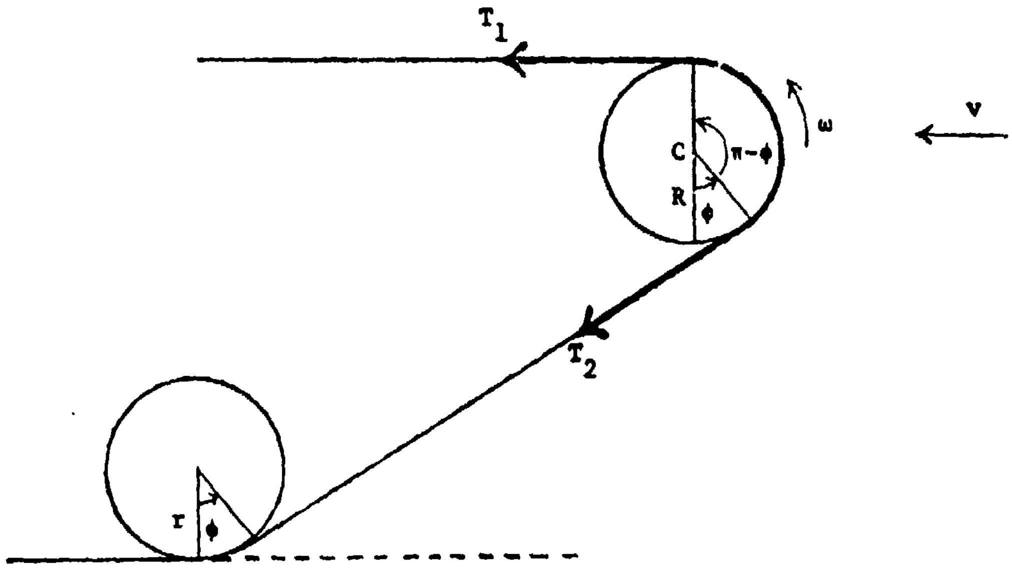


Figure 6. Tension forces acting on the idler wheel support.

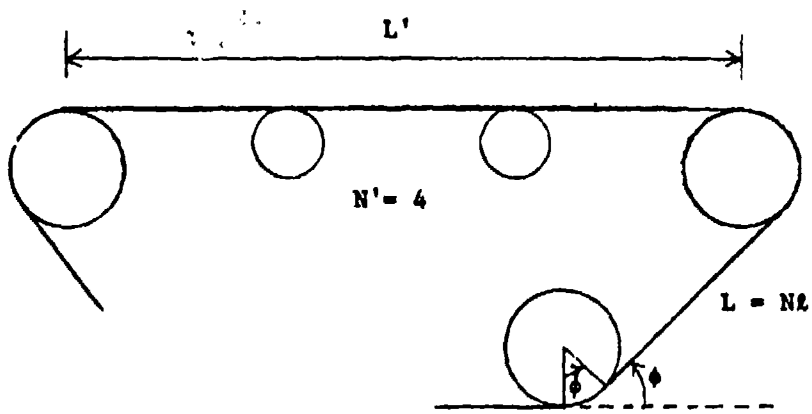


Figure 7. Illustration of a track configuration.

These mass contributions may be approximated by

$$M = M_I + \left(\frac{L'}{N'}\right)m + (Nm)\sin\phi, \tag{45}$$

where

N' = number of support rollers + 2(for idler wheel and sprocket),

N = number of track shoe assemblies in L,

m = mass of a shoe assembly.

The factor $\sin\phi$ in the third term adjusts Nm contribution for the angle the track assumes with the horizontal between the rear road wheel and the idler (See Figure 7). Again, this static force does not directly contribute to the vibration coupling between the track and the vehicle compartment.

5.3 Centripetal Force on the Track

Figure 8 below illustrates the idler wheel and the part of track wrapping around it.

At first, assume a constant speed motion. Then the centripetal force on the track in contact with the wheel is given by

$$F_c = \int_{-\frac{(\pi-\phi)}{2}}^{\frac{\pi-\phi}{2}} (\lambda R\, d\theta)\frac{v^2}{R}\cos\theta = \lambda v^2 \sin\theta \Big|_{-\frac{(\pi-\phi)}{2}}^{\frac{\pi-\phi}{2}}$$

$$= 2\sin\left(\frac{\pi}{2} - \frac{\phi}{2}\right) = 2\lambda v^2\cos(\phi/2), \qquad 0<\phi<\frac{\pi}{2}. \tag{46}$$

Note that F_c depends both on v and ϕ.

The centripetal force F_c is maximum at $\phi=0$, but in the fore-aft direction. It is minimum at $\phi=\pi/2$, but not zero, and is directed in the

45° from the vertical. Therefore, even though the magnitude of F_c is maximum when $\phi \to 0$, the angle ϕ must be made to approach zero if the transverse component of the force is to be made small.

For a fixed angle ϕ, if the angular velocity of the track around the idler wheel changes, the corresponding change in F_c is given by

$$\frac{dF_c}{dt} = 4\lambda v \cos\left(\frac{\phi}{2}\right) \frac{dv}{dt} = 4\lambda v a \cos\left(\frac{\phi}{2}\right), \tag{47}$$

where $a = dv/dt$ = linear acceleration.

As the vehicle accelerates, decelerates or changes direction, thereby causing the change in the angular velocity of the track rounding the idler wheel, the interaction force between the track and the idler wheel also changes in time and it will be transmitted to the vehicle compartment.

To minimize this contribution of dynamic effect in the transverse direction, the angle ϕ must be made as small as possible.

5.4 Net Force Applied on the Idler Wheel Support

Combining the forces described in the preceding three sections, one obtains the components of net force applied on the idler wheel support as shown in Figure 9. They are

$$\begin{aligned} F_x &= -T - T\cos\phi - F_c\cos(\phi/2), \\ F_y &= -Mg - T\sin\phi - F_c \sin(\phi/2). \end{aligned} \tag{48}$$

The transverse component of forces will be a minimum for the minimum value of F_y. This condition will be approached as $\phi \to 0$. In practice, a small value of ϕ can probably be realized by making the idler wheel diameter large.

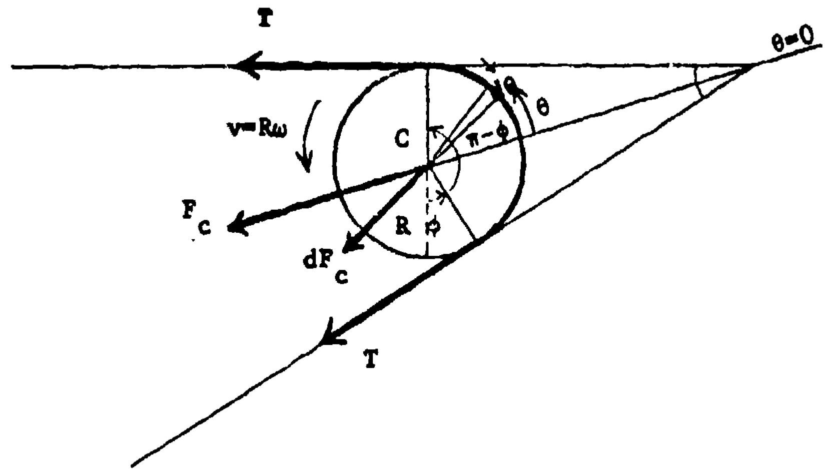


Figure 8. Analysis of centripetal force on the track going around the idler wheel.

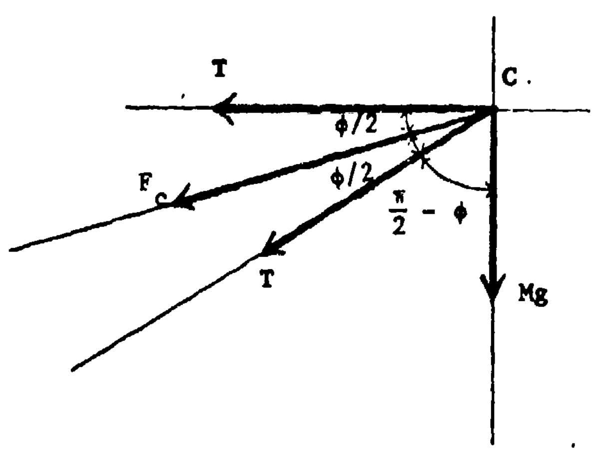


Figure 9. Analysis of net force applied on the idler wheel support.

VI CONCLUSION

The transfer receptance calculation carried out in 4.3.2 yields the ratio u_B/F_0 which shows the magnitude of displacement at the idler wheel caused by the force applied at the rear road wheel. It is assumed that this force is a periodic one whose frequency is directly proportional to the speed of the vehicle, that is to the frequency with which the track shoe assembly strikes the ground.

A previous study* shows that the crew members of tracked vehicles are affected adversely by low frequency vibrations transmitted to the vehicle compartment through the idler suspension. In particular, it is the transverse mode of these vibrations, rather than the fore-aft component, that needs to be reduced for the comfort of the crew as well as to minimize the energy loss arising from the chordal action in the track.

The expression for the ratio u_B/F_0 as shown in equation (36) yields the following result: In order to reduce this ratio

(a) the compliance of the idler suspension should be relatively stiff. this can be achieved in practice by making the receptance at the idler wheel position small, but not zero corresponding to the fixed-end boundary condition, and making the mass of the idler wheel large.

(b) The size of each shoe assembly should be small. The continuous-belt limit is an optimum limit.

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*Report No. MPG-178, "Noise and Vibration Reduction Program on the M-109 Self-Propelled Howitzer", Allison Division Army Tank-Automotive Plant, 1966.

Analysis of dynamic forces acting on the idler wheel support, as discussed in 5.4, shows that the transverse component of the vibration in the part of the track between the rear road wheel and the idler wheel will be minimized by

(c) making the angle as small as possible that this part of the track makes with the horizontal. In practice, this condition may be achieved by making the diameter of the idler wheel large.

The combination of the criteria described in (a), (b), and (c) above leads one to the condition that would reduce the transverse components of the vibration transmitted to the vehicle compartment through the idler suspension: The mass and the size of the idler wheel should be large and the size of individual track shoe assembly should be small. The compliance of the idler suspension should be fairly stiff.

The ratio u_B/F_0 is expressed in terms of various physical parameters, as shown in equations (36) and (37). This is not a smooth function and one must resort to the numerical analysis to study its characteristics. Nevertheless, such an analysis enables one to obtain a set of values for various physical parameters that will result in small magnitude of u_B for a given force input at the road wheel. Several typical cases are illustrated in Appendix A. The result shows that there is no single suspension system that will reduce vibration coupling at all frequencies.

Similar calculations as those shown in Appendix A may be carried out for many other cases. The calculations are simple enough to be performed in a hand-held calculator.

VII RECOMMENDATION FOR FURTHER STUDY

The method of analysis based on the technique of receptance coefficients is a straightforward and general one that can be applied to many vibration problems.

The present study dealt only with an idealized model of a single-pin type track configuration. The study should be extended to deal with other types of track configuration, such as the double-pin type.

No theoretical analysis can be regarded as complete without a follow-up study of experimental verification of its results. It is recommended that some or all aspects of the present study be subjected to an experimental project for verification.

APPENDIX A: SAMPLE CALCULATIONS OF u_B/F_O

TABLE 1. $\sin\epsilon = [\frac{I\omega^2}{K} - (\frac{I\omega^2}{2K})^2]^{1/2}$

$\frac{I\omega^2}{K}$	sin ε	sin 5ε	sin 6ε	sin 7ε	sin 8ε	sin 9ε	sin 10ε
0.25 3.75	0.4841	0.5768	0.1094	-0.3855	-0.7840	-0.9865	-0.9424
0.50 3.50	0.6614	-0.4545	-0.9300	-0.9406	-0.4809	0.2192	0.8097
0.75 3.25	0.7806	-0.9727	-0.7892	-0.0138	0.7719	0.9787	0.4516
1.00 3.00	0.8660	-0.8660	0.0000	0.8660	0.8660	0.0000	-0.8660
1.25 2.75	0.9270	-0.3443	0.7412	0.9003	-0.0658	-0.9497	-0.6465
1.50 2.50	0.9682	0.3017	0.9986	0.1979	-0.8995	-0.6480	0.5753
1.75 2.25	0.9922	0.8100	0.6816	-0.6411	-0.8414	0.4313	0.9489
2.00	1.0000	1.0000	0.0000	-1.0000	0.0000	1.0000	0.0000

$$A = (1 - \alpha')\sin N\epsilon - \sin(N+1)\epsilon,$$

$$B = -(1/\ell^2\alpha_B)u_B/F_0 = (\sin\epsilon)/A,$$

as used in the following tables. Also refer to equations (43) and (44).

TABLE 2. $\alpha' = 0.01$

$\frac{I\omega^2}{K}$	A(N=5)	A(N=6)	A(N=7)	A(N=8)	A(N=9)
0.25	0.4616	0.4938	0.4024	0.2103	-0.0342
0.50	0.4798	0.0199	-0.4503	-0.6953	-0.5927
0.75	1.7522	-0.7675	-0.7856	-0.2145	0.5173
1.00	-0.8573	-0.8660	-0.0087	0.8573	0.8660
1.25	-1.0821	-0.1665	0.9571	1.0148	-0.2937
1.50	-0.6990	0.7907	1.0954	-0.2425	-1.2168
1.75	0.1203	1.3159	0.2067	-1.2643	-0.5219
2.00	0.9900	1.0000	-0.9900	-1.0000	0.9900

	B(N=5)	B(N=6)	B(N=7)	B(N=8)	B(N=9)
0.25	1.0487	0.9804	1.2030	2.2728	-14.1550
0.50	1.3785	33.2362	-1.4688	-0.9512	-1.1159
0.75	0.4455	-1.0171	-0.9936	-3.6392	1.5090
1.00	-1.0101	-1.0000	-99.5402	1.0101	1.0000
1.25	-0.8567	-5.5676	0.9686	0.9135	-3.1563
1.50	-1.3851	1.2245	0.8839	-3.9926	-0.7957
1.75	8.2477	0.7540	4.8002	-0.7848	-1.9011
2.00	1.0101	1.0000	-1.0101	-1.0000	1.0101

	$\frac{B}{N^2}$(N=5)	$\frac{B}{N^2}$(N=6)	$\frac{B}{N^2}$(N=7)	$\frac{B}{N^2}$(N=8)	$\frac{B}{N^2}$(N=9)
0.25	0.0419	0.0272	0.0246	0.0355	-0.1748
0.50	0.0551	0.9232	-0.0300	-0.0149	-0.0138
0.75	0.0178	-0.0283	-0.0203	-0.0569	0.0186
1.00	-0.0404	-0.0278	-2.0314	0.0158	0.0123
1.25	-0.0343	-0.1547	0.0198	0.0143	-0.0390
1.50	-0.0554	0.0340	0.0180	-0.0624	-0.0098
1.75	0.3299	0.0209	0.0980	-0.0123	-0.0235
2.00	0.4040	0.0278	-0.0206	-0.0156	0.0125

TABLE 3. $\alpha' = 0.05$

$\frac{I\omega^2}{K}$	A(N=5)	A(N=6)	A(N=7)	A(N=8)	A(N=9)
0.25	0.4386	0.4894	0.4178	0.2417	0.0052
0.50	0.4980	0.0571	-0.4127	-0.6761	-0.6015
0.75	-0.1349	-0.7359	-0.7850	-0.2454	0.4782
1.00	-0.8227	-0.8660	-0.0433	0.8227	0.8660
1.25	-0.0683	-0.1962	0.9211	0.8872	-0.2557
1.50	-0.7111	0.7501	1.0875	-0.2065	-1.1909
1.75	0.0879	1.2886	0.2324	-1.2306	-0.5392
2.00	0.9500	1.0000	-0.9500	-1.0000	0.9500

	B(N=5)	B(N=6)	B(N=7)	B(N=8)	B(N=9)
0.25	1.1037	0.9892	1.1587	2.0029	93.0962
0.50	1.3281	11.5832	-1.6026	-0.9783	-1.0996
0.75	-5.7865	-1.0607	-0.9944	-3.1809	1.6324
1.00	-1.0526	-1.0000	-20.0000	1.0526	1.0000
1.25	-0.8677	-4.7248	1.0064	1.0449	-3.6253
1.50	-1.3616	1.2908	0.8903	-4.6886	-0.8130
1.75	11.2878	0.7700	4.2694	-0.8063	-1.8401
2.00	1.0526	1.0000	-1.0526	-1.0000	1.0526

	$\frac{B}{N^2}$(N=5)	$\frac{B}{N^2}$(N=6)	$\frac{B}{N^2}$(N=7)	$\frac{B}{N^2}$(N=8)	$\frac{B}{N^2}$(N=9)
0.25	0.0441	0.0275	0.0236	0.0313	1.1493
0.50	0.0531	0.3218	-0.0327	-0.0153	-0.0136
0.75	-0.2315	-0.0295	-0.0203	-0.0497	0.0202
1.00	-0.0421	-0.0278	-0.4082	0.0164	0.0123
1.25	-0.0347	-0.1312	0.0205	0.0163	-0.0448
1.50	-0.0545	0.0359	0.0182	-0.0733	-0.0100
1.75	0.4515	0.0214	0.0871	-0.0126	-0.0227
2.00	0.0421	0.0278	-0.0215	-0.0156	0.0130

TABLE 4. $\alpha' = 0.10$

$\frac{I\omega^2}{K}$	A(N=5)	A(N=6)	A(N=7)	A(N=8)	A(N=9)
0.25	0.4097	0.4840	0.4371	0.2809	0.0546
0.50	0.5210	0.1036	-0.3656	-0.6520	-0.6124
0.75	-0.0862	-0.6965	-0.7843	-0.2840	0.4292
1.00	-0.7784	-0.8660	-0.0866	0.7794	0.8660
1.25	-1.0511	-0.2332	0.8761	0.8905	-0.2082
1.50	-0.7271	0.7008	1.0776	-0.1616	-1.1585
1.75	0.0474	1.2545	0.2644	-1.1886	-0.5617
2.00	0.9000	1.0000	-0.9000	-1.0000	0.9000

	B(N=5)	B(N=6)	B(N=7)	B(N=8)	B(N=9)
0.25	1.1816	1.0002	1.1075	1.7234	8.8663
0.50	1.2695	6.3842	-1.8091	-1.0144	-1.0800
0.75	-9.0557	-1.1207	-0.9953	-2.7486	1.8187
1.00	-1.1125	-1.0000	-10.0000	1.1111	1.0000
1.25	-0.8819	-3.9751	1.0581	1.0410	-4.4524
1.50	-1.3316	1.3816	0.8985	-5.9913	-0.8357
1.75	20.9325	0.7909	3.7526	-0.8348	-1.7664
2.00	1.1111	1.0000	-1.1111	-1.0000	1.1111

	$\frac{B}{N^2}$(N=5)	$\frac{B}{N^2}$(N=6)	$\frac{B}{N^2}$(N=7)	$\frac{B}{N^2}$(N=8)	$\frac{B}{N^2}$(N=9)
0.25	0.0473	0.0278	0.0226	0.0269	0.1095
0.50	0.0508	0.1773	-0.0369	-0.0159	-0.0133
0.75	-0.3622	-0.0311	-0.0203	-0.0429	0.0225
1.00	-0.0445	-0.0278	-0.2041	0.0174	0.0123
1.25	-0.0353	-0.1104	0.0216	0.0163	-0.0550
1.50	-0.0533	0.0384	0.0183	-0.0936	-0.0103
1.75	0.8373	0.0220	0.0766	-0.0130	-0.0218
2.00	0.0444	0.0278	-0.0227	-0.0156	0.0137

TABLE 5. $\alpha' = 0.15$

$\frac{I\omega^2}{K}$	A(N=5)	A(N=6)	A(N=7)	A(N=8)	A(N=9)
0.25	0.3809	0.4785	0.4563	0.3201	0.1039
0.50	0.5437	0.1501	-0.3186	-0.6280	-0.6234
0.75	-0.0376	-0.6570	-0.7836	-0.3226	0.3803
1.00	-0.7361	-0.8660	-0.1299	0.7361	0.8660
1.25	-1.0339	-0.2703	0.8311	0.8938	-0.1607
1.50	-0.7422	0.6509	1.0677	-0.1166	-1.1261
1.75	0.0069	1.2205	0.2965	-1.1465	-0.5823
2.00	0.8500	1.0000	-0.8500	-1.0000	0.8500

	B(N=5)	B(N=6)	B(N=7)	B(N=8)	B(N=9)
0.25	1.2709	1.0117	1.0609	1.5123	4.6593
0.50	1.2165	4.4064	-2.0760	1.0532	-1.0610
0.75	-20.7606	-1.1881	-0.9962	-2.4197	2.0526
1.00	-1.1765	-1.0000	-6.6667	1.1765	1.0000
1.25	-0.8966	-3.4295	1.1154	1.0371	-5.7685
1.50	-1.3045	1.4875	0.9068	-8.3036	-0.8598
1.75	143.7971	0.8129	3.3464	-0.8654	-1.7039
2.00	1.1765	1.0000	-1.1765	-1.0000	1.1765

	$\frac{B}{N^2}$(N=5)	$\frac{B}{N^2}$(N=6)	$\frac{B}{N^2}$(N=7)	$\frac{B}{N^2}$(N=8)	$\frac{B}{N^2}$(N=9)
0.25	0.0508	0.0281	0.0217	0.0236	0.0575
0.50	0.0487	0.1224	-0.0424	-0.0165	-0.0131
0.75	-0.8304	-0.0330	-0.0203	-0.0378	0.0253
1.00	-0.0471	-0.0278	-0.1361	0.0184	0.0123
1.25	-0.0359	-0.0953	0.0228	0.0162	-0.0712
1.50	-0.0522	0.0413	0.0185	-0.1297	-0.0106
1.75	5.7519	0.0226	0.0683	-0.0135	-0.0210
2.00	0.0471	0.0278	-0.0240	-0.0156	0.0145

TABLE 6. $\alpha' = 0.20$

$\frac{I\omega^2}{K}$	A(N=5)	A(N=6)	A(N=7)	A(N=8)	A(N=9)
0.25	0.3520	0.4730	0.4756	0.3593	0.1532
0.50	0.5664	0.1966	-0.2716	-0.6039	-0.6343
0.75	0.0110	-0.6176	-0.7829	-0.3612	0.3314
1.00	-0.6928	-0.8660	-0.1732	0.6928	0.8660
1.25	-1.0166	-0.3073	0.7860	0.8971	-0.1133
1.50	-0.7572	0.6010	1.0578	-0.0716	-1.0937
1.75	-0.0336	1.1864	0.3285	-1.1044	-0.6039
2.00	0.8000	1.0000	-0.8000	-1.0000	0.8000

	B(N=5)	B(N=6)	B(N=7)	B(N=8)	B(N=9)
0.25	1.3753	1.0235	1.0179	1.3473	3.1599
0.50	1.1677	3.3642	-2.4352	-1.0952	-1.0427
0.75	70.9637	-1.2639	-0.9971	-2.1611	2.3555
1.00	-1.2500	-1.0000	-5.0000	1.2500	1.0000
1.25	-0.9119	-3.0166	1.1794	1.0333	-8.1818
1.50	-1.2787	1.6110	0.9153	-13.5223	-0.8853
1.75	-29.5298	0.8363	3.0204	-0.8984	-1.6430
2.00	1.2500	1.0000	-1.2500	-1.0000	1.2500

	$\frac{B}{N^2}$(N=5)	$\frac{B}{N^2}$(N=6)	$\frac{B}{N^2}$(N=7)	$\frac{B}{N^2}$(N=8)	$\frac{B}{N^2}$(N=9)
0.25	0.0550	0.0284	0.0208	0.0211	0.0390
0.50	0.0467	0.0934	-0.0497	-0.0171	-0.0129
0.75	2.8385	-0.0351	-0.0203	-0.0338	0.0291
1.00	-0.0500	-0.0278	-0.1020	0.0195	0.0123
1.25	-0.0365	-0.0838	0.0241	0.0161	-0.1010
1.50	-0.0511	0.0447	0.0187	-0.2113	-0.0109
1.75	-1.1812	0.0232	0.0616	-0.0140	-0.0203
2.00	0.0500	0.0278	-0.0255	-0.0278	0.0500

TABLE 7. $\alpha' = 0.25$

$\frac{I\omega^2}{K}$	A(N=5)	A(N=6)	A(N=7)	A(N=8)	A(N=9)
0.25	0.3232	0.4676	0.4949	0.3985	0.2025
0.50	0.5891	0.2431	-0.2246	-0.5799	-0.6453
0.75	0.0597	-0.5781	-0.7823	-0.3998	0.2824
1.00	-0.6495	-0.8660	-0.2165	0.6495	0.8660
1.25	-0.9994	-0.3444	0.7410	0.9004	-0.0658
1.50	-0.7723	0.5511	1.0479	-0.0266	-1.0613
1.75	-0.0741	1.1523	0.3606	-1.0624	-0.6254
2.00	0.7500	1.0000	-0.7500	-1.0000	0.7500

	B(N=5)	B(N=6)	B(N=7)	B(N=8)	B(N=9)
0.25	1.4978	1.0353	0.9782	1.2148	2.3906
0.50	1.1227	2.7207	-2.9448	-1.1405	-1.0249
0.75	13.0754	-1.3503	-0.9978	-1.9525	2.7642
1.00	-1.3333	-1.0000	-4.0000	1.3333	1.0000
1.25	-0.9276	-2.6916	1.2510	1.0295	-14.0881
1.50	-1.2536	1.7568	0.9239	-36.3985	-0.9123
1.75	-13.3900	0.8611	2.7515	-0.9339	-1.5865
2.00	1.3333	1.0000	-1.3333	-1.0000	1.3333

	$\frac{B}{N^2}$(N=5)	$\frac{B}{N^2}$(N=6)	$\frac{B}{N^2}$(N=7)	$\frac{B}{N^2}$(N=8)	$\frac{B}{N^2}$(N=9)
0.25	0.0599	0.0288	0.0200	0.0190	0.0295
0.50	0.0449	0.0756	-0.0601	-0.0178	-0.0127
0.75	0.5230	-0.0375	-0.0204	-0.0305	0.0341
1.00	-0.0533	-0.0278	-0.0816	0.0208	0.0123
1.25	-0.0371	-0.0748	0.0255	0.0161	-0.1739
1.50	-0.0501	0.0488	0.0189	-0.5687	-0.0113
1.75	-0.5356	0.0239	0.0562	-0.0146	-0.0196
2.00	0.0533	0.0278	-0.0272	-0.0156	0.0165

These tables may be used in the following manner:

Assuming that a track shoe assembly has a characteristic value for I/K, the selection of angular frequency ω will fix the value of $I\omega^2/K$ for the particular frequency of vibration that needs to be minimized.

(a) Assume that the boundary condition at the idler wheel is pre-determined, that is the value of α' is given. Then the calculations similar to those tabulated will give the value of N which would minimize the particular vibration.

(b) Assume that N is pre-determined, that is, the distance between the rear road wheel and the idler wheel is a fixed one. Then the calculations similar to those tabulated will give the value of α' which would minimize the particular vibration.

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